Photometric Stereo Reconstruction Based on Sparse Representation

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May 17, 2016

Overview

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Motivation

- Monocular vision versus binocular vision
- Basic photometric stereo: only ideal Lambertian diffuse model
  - Considering non-diffuse corruption (e.g. shadows and specularities)
  - Sparsity
  - Extending to non-Lambertian diffuse reflection (e.g. specular reflection and mixed reflection)

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 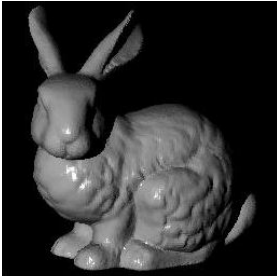 

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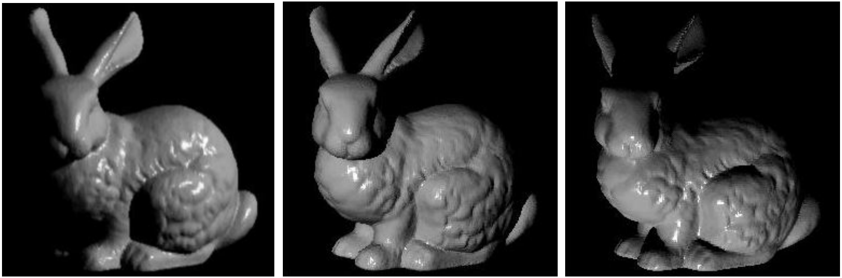

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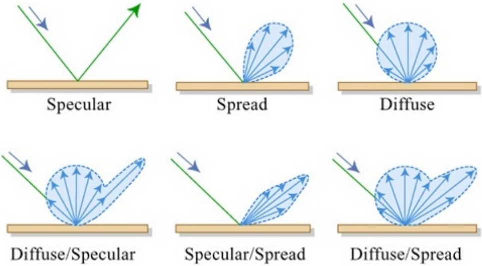

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- Procedure:
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    - Depth estimation
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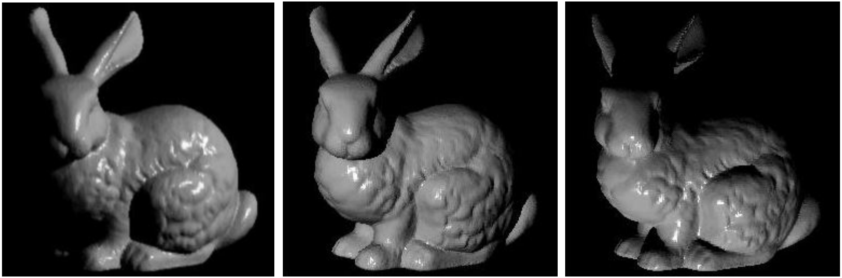

Basic Photometric Stereo: Woodham's Method

Recovery of Surface Normal

- Recovering surface normal while ignoring non-diffuse corruption
- **Lambertian diffuse model:**

$$\boldsymbol{I} = \rho \boldsymbol{n}^T \boldsymbol{l}$$

- where
  - $\boldsymbol{n} \in \mathbb{R}^3$ is the surface normal at the point
  - $\boldsymbol{I} \in \mathbb{R}^{1xm}$ is the observed intensity at this point in m images
  - $\boldsymbol{l} \in \mathbb{R}^{3xm}$ is the incoming lighting direction in m images
  - $\rho \in \mathbb{R}$ is the diffuse albedo
- Degree of freedom: 4
- Given 4 images, n can be recovered via solving linear system
- **Given more than 4 images($m > 4$): least square (LS)**
  - Close-form solution according to normal equation:

$$\boldsymbol{n}^T = \boldsymbol{I} \cdot \boldsymbol{l}^T \cdot (\boldsymbol{l}\boldsymbol{l}^T)^{-1} \rho^{-1}$$

- **Limitation**: non-Lambertian diffuse reflection, non-diffuse corruption

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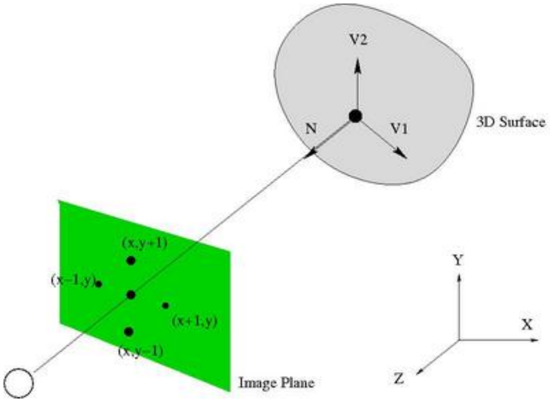
Basic Photometric Stereo: Woodham's Method

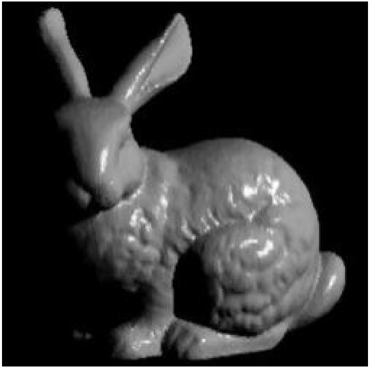
Depth Estimation

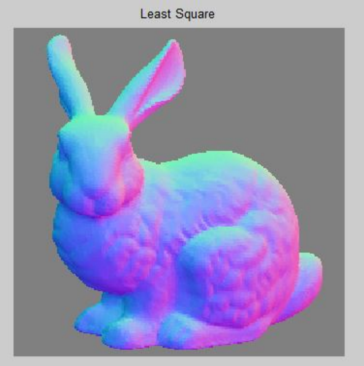
- Surface normal is approximately perpendicular to the vector formed with adjacent pixel
- Least square again



Basic Photometric Stereo: Woodham's Method

Reconstruction Result





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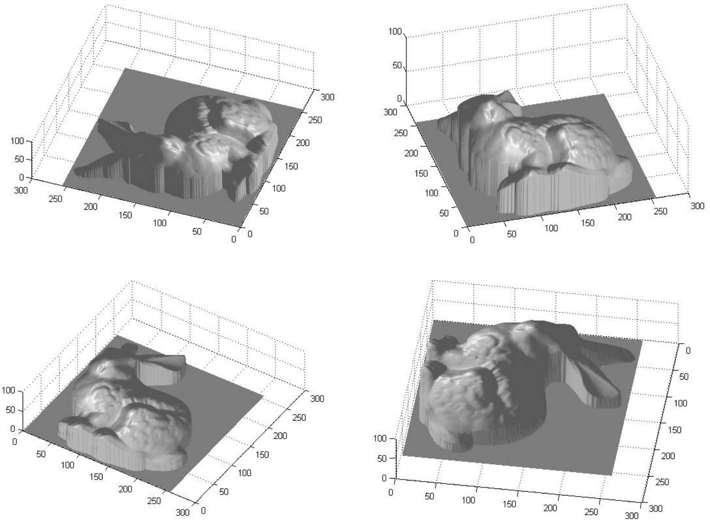

Overview

- Motivation: introduce a parameter to interpret corruption
- New reflectance model:

$$\boldsymbol{I} = \rho \boldsymbol{n}^T \boldsymbol{l} + \boldsymbol{e}$$

- where
  - $\boldsymbol{e} \in \mathbb{R}^{1xm}$ is the additive corruption at this point in m images
- **Degree of freedom:** $m + 4 > m$ forever, unconstrained problem
- **Property:** exhibiting dominant diffuse reflection while non-diffuse effects emerge primarily in limited areas $\Longrightarrow$ $\boldsymbol{e}$ is sparse
- Reformulating as following

$$\min_{\boldsymbol{x},\boldsymbol{e}} \|\boldsymbol{e}\|_0 \quad s.t. \quad \boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{e}$$

- With relaxation for compensate more modeling errors

$$\min_{\boldsymbol{x},\boldsymbol{e}} \|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x} - \boldsymbol{e}\|_2^2 + \lambda \|\boldsymbol{e}\|_0$$

Modeling with Sparse Regularized Algorithms

Representation of Non-Diffuse Corruption

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- Model restatement

$$\min_{\boldsymbol{x},\boldsymbol{e}} \|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x} - \boldsymbol{e}\|_2^2 + \lambda\|\boldsymbol{e}\|_0$$

- where
    - λ is a nonnegative trade-off parameter
    - $\|\cdot\|_0$ represents the ℓ_0-norm
- However, ℓ_0-norm is discontinuous and non-convex
- **Replacing ℓ_0-norm with ℓ_1-norm**

$$\min_{\boldsymbol{x},\boldsymbol{e}} \|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x} - \boldsymbol{e}\|_2^2 + \lambda\|\boldsymbol{e}\|_1$$

- ℓ_1-norm is convex, easy to solve
    - Sequentially added sign constraints algorithm
    - Non-negative variables method
    - Iterated ridge regression (IRR), hybrid of lasso and ridge regression
- **Limitation**: approximation from ℓ_0-norm to ℓ_1-norm, non-Lambertian diffuse reflection

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Lasso Sparse Regression (L1) Model

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Representation of Sparse Bayesian Learning (SBL) Model

Bayesian Inference

- Motivation: simple hierarchical Bayesian model to estimate $\boldsymbol{x}, \boldsymbol{e}$
- Likelihood function

$$p(\boldsymbol{y}|\boldsymbol{x}, \boldsymbol{e}) = \mathcal{N}(\boldsymbol{y}; \boldsymbol{A}\boldsymbol{x} + \boldsymbol{e}, \lambda \boldsymbol{I})$$

- Conjugate prior

$$p(\boldsymbol{x}) = \mathcal{N}(\boldsymbol{x}; \boldsymbol{0}, \sigma_x^2 \boldsymbol{I})$$
$$p(\boldsymbol{e}) = \mathcal{N}(\boldsymbol{e}; \boldsymbol{0}, \boldsymbol{\Gamma})$$

- Posterior

$$p(\boldsymbol{x}, \boldsymbol{e}|\boldsymbol{y}) \propto p(\boldsymbol{y}|\boldsymbol{x}, \boldsymbol{e}) p(\boldsymbol{x}) p(\boldsymbol{e})$$

- Marginal posterior

$$p(\boldsymbol{x}|\boldsymbol{y}) = \int p(\boldsymbol{x}, \boldsymbol{e}|\boldsymbol{y}) d\boldsymbol{e} = \mathcal{N}(\boldsymbol{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma})$$

- With mean and covariance calculated as

$$\boldsymbol{\mu} = \boldsymbol{\Sigma} \boldsymbol{A}^T (\boldsymbol{\Gamma} + \lambda \boldsymbol{I})^{-1} \boldsymbol{y}$$
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Representation of Sparse Bayesian Learning (SBL) Model

Majorization-Minimization Approach

- Motivation: estimating $\mathbf{\Gamma}$ accurately based on data itself
- Likelihood function of $\mathbf{\Gamma}$:
  - Empirical Bayesian approach

$$L(\mathbf{\Gamma}) \triangleq \int p(\boldsymbol{y}|\boldsymbol{x}, \boldsymbol{e})p(\boldsymbol{x})p(\boldsymbol{e})d\boldsymbol{e}d\boldsymbol{x} = \mathcal{N}(\boldsymbol{y}; \mathbf{0}, \mathbf{\Sigma}_y)$$

  - where

$$\mathbf{\Sigma}_y = \sigma_x^2 \boldsymbol{A}\boldsymbol{A}^T + \mathbf{\Gamma} + \lambda \boldsymbol{I}$$

- Maximum likelihood estimation

$$\mathbf{\Gamma} = \arg\max L(\mathbf{\Gamma})$$

- Equivalently, we should minimize

$$L(\mathbf{\Gamma}) \triangleq -\ln \int p(\boldsymbol{y}|\boldsymbol{x}, \boldsymbol{e})p(\boldsymbol{x})p(\boldsymbol{e})d\boldsymbol{e}d\boldsymbol{x}$$
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- Problem restatement

$$\min_{\boldsymbol{\Gamma}} L(\boldsymbol{\Gamma}) = \min_{\boldsymbol{\Gamma}} (\ln|\boldsymbol{\Sigma}_y| + \boldsymbol{y}^T \boldsymbol{\Sigma}_y^{-1} \boldsymbol{y})$$
$$\boldsymbol{\Sigma}_y = \sigma_x^2 \boldsymbol{A}\boldsymbol{A}^T + \boldsymbol{\Gamma} + \lambda \boldsymbol{I}$$

- **Solution: constructing an upper bound and iteratively tightening it**
- For brevity, using results directly from thesis
  - Suppose Γ is fixed, partitioning problem into two parts

$$\ln|\boldsymbol{\Sigma}_y| \leq \sum_i (\frac{u_i}{\gamma_i} + \ln \gamma_i) - h^*(\boldsymbol{u})$$
$$\boldsymbol{y}^T \boldsymbol{\Sigma}_y^{-1} \boldsymbol{y} \leq \sum_i \frac{z_i^2}{\gamma_i} + f(\boldsymbol{z})$$

  - Equalities are obtained if and only if

$$\boldsymbol{u} = diag[(\sigma_x^2 \boldsymbol{A}\boldsymbol{A}^T + \lambda \boldsymbol{I})^{-1} + \boldsymbol{\Gamma}^{-1}]^{-1}$$
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Representation of Sparse Bayesian Learning (SBL) Model

Majorization-Minimization Approach (cont.)

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- **Solution: constructing an upper bound and iteratively tightening it**
- For brevity, using results directly from thesis
  - Suppose $\boldsymbol{u}, \boldsymbol{z}$ are fixed

$$L(\mathbf{\Gamma}) \leq \sum_i (\frac{u_i + z_i^2}{\gamma_i} + \ln \gamma_i) + Constant$$

  - Equality is obtained if and only if

$$\gamma_i = u_i + z_i^2$$

$$\mathbf{\Gamma} = diag[\gamma]$$

Representation of Sparse Bayesian Learning (SBL) Model

Majorization-Minimization Approach (cont.)

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Majorization-minimization approach

1. initialize $\boldsymbol{\Gamma}^0$
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6. end while

- **Limitation**: non-Lambertian diffuse reflection

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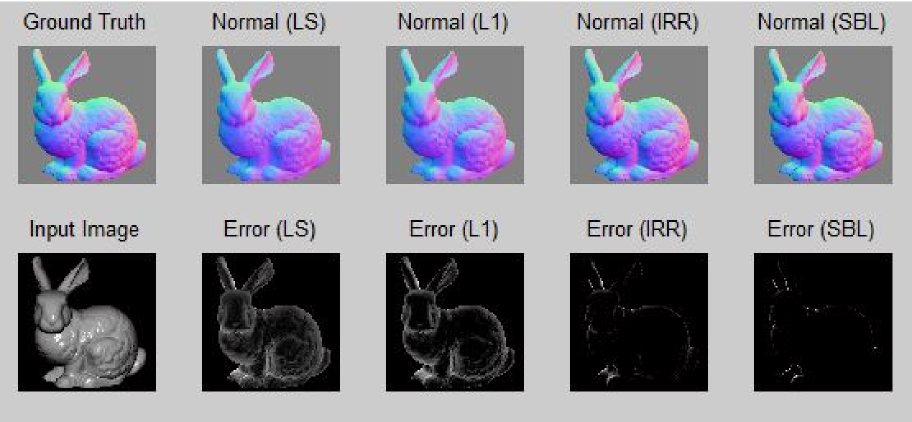
- Recovery of surface normal and error map
- Noise is 10 percent
- Shadows and specularities are rendered (same below)



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$$p(\boldsymbol{y}|\boldsymbol{x}, \boldsymbol{e}) = \mathcal{N}(\boldsymbol{y}; \boldsymbol{A}\boldsymbol{x} + \boldsymbol{e}, \lambda \boldsymbol{I})$$

- All above are based on Lambertian diffuse model
- What if more mixed and complicated materials?

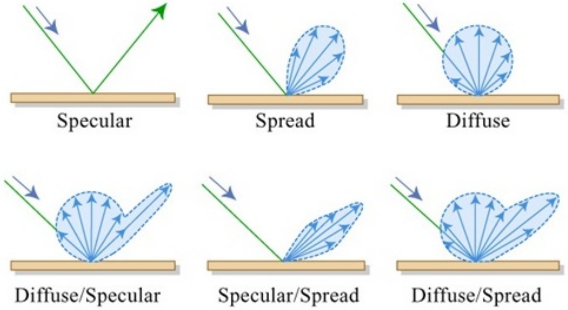
- Model Restatement

$$\boldsymbol{I} = \rho \boldsymbol{n}^T \boldsymbol{l} + \boldsymbol{e}$$

$$\min_{\boldsymbol{x},\boldsymbol{e}} \|\boldsymbol{e}\|_0 \quad s.t. \quad \boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{e}$$

$$p(\boldsymbol{y}|\boldsymbol{x},\boldsymbol{e}) = \mathcal{N}(\boldsymbol{y}; \boldsymbol{A}\boldsymbol{x} + \boldsymbol{e}, \lambda \boldsymbol{I})$$

- All above are based on Lambertian diffuse model
- What if more mixed and complicated materials?



Extension with Piecewise Linear Model

Linear Combination Model

- Extension Lambertian model to general function

$$\boldsymbol{I} = f(\boldsymbol{n}^T \boldsymbol{l})$$

- Monotonicity

$$\boldsymbol{n}^T \boldsymbol{l}_i > \boldsymbol{n}^T \boldsymbol{l}_j \leftrightarrow f(\boldsymbol{n}^T \boldsymbol{l}_i) > f(\boldsymbol{n}^T \boldsymbol{l}_j)$$

- Inverse diffuse reflectance model

$$f^{-1}(\boldsymbol{I}) = g(\boldsymbol{I}) = \boldsymbol{n}^T \boldsymbol{l}$$

- Given the linearity on right-hand-side, representing $g(\cdot)$ as

$$g(\boldsymbol{I}) = \sum_{k=1}^{p} a_k g_k(\boldsymbol{I})$$

- $\boldsymbol{a}$ is an unknown weight vector, p is set as the number of segments
- $g_k(\boldsymbol{I})$ is non-linear basis function, multiple choices for $g_k(\boldsymbol{I})$:
  - polynomial, Gaussian, logistic, piecewise linear and spline function

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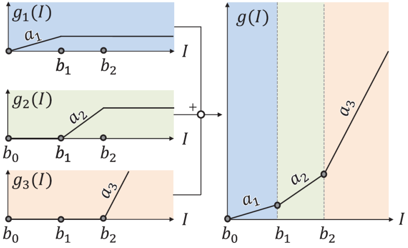
- For modest computational cost, representing $g_k(\boldsymbol{I})$ with piecewise linear function

$$g_k(I_j) = \begin{cases} 0 & 0 \leq I_j < b_{k-1} \\ I_j - b_{k-1} & b_{k-1} \leq I_j < b_k \\ b_k - b_{k-1} & b_k \leq I_j \end{cases}$$



- b_k are segment points, equally separate the range of I and $b_0 = 0$
- $g(\boldsymbol{I})$ is continuous and monotonically increasing, intersect the origin

Extension with Piecewise Linear Model

Piecewise Linear Model

- With linear combination model and piecewise linear function

$$\boldsymbol{n}^T \boldsymbol{l} - \sum_{k=1}^{p} a_k g_k(\boldsymbol{I}) = 0$$

- For avoiding the degenerate $x = 0$ solution by constraining

$$\sum_{k=1}^{p} a_k = 1$$

- Representing above as

$$\boldsymbol{A}^* \boldsymbol{x} = \boldsymbol{y}^*$$

- $\boldsymbol{x}$ is an unknown vector, $\boldsymbol{x} \triangleq [n_x, n_y, n_z, a_1, a_2, \ldots, a_p]^T \in \mathbb{R}^{p+3}$
- **Piecewise linear least square (PL-LS) model, piecewise linear sparse Bayesian learning (PL-SBL) model**
- Results closely match ground truth!

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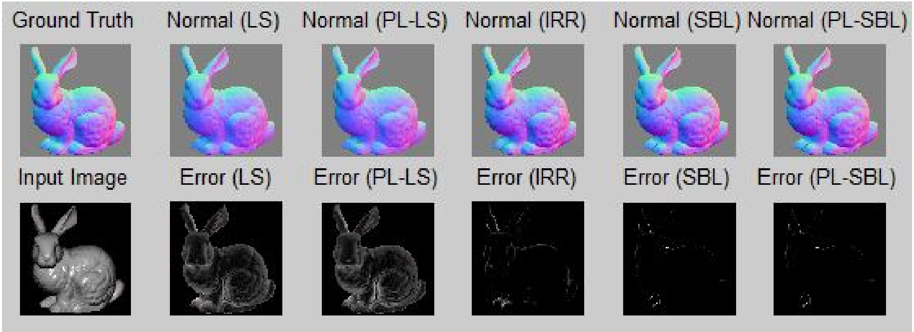
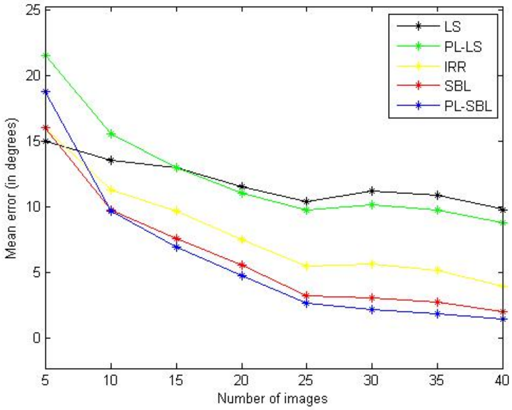
Extension with Piecewise Linear Model

Experimental Results and Analysis

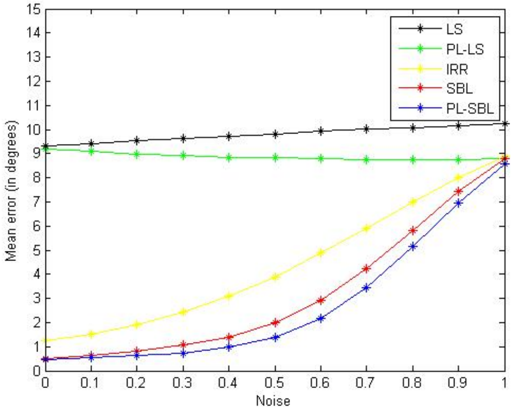
- Recovery of surface normal and error map
- Noise is 10 percent
- Best p is set for PL-LS ($p = 2$) and PL-SBL ($p = 4$)
- Best σ_a^2 is set for PL-SBL



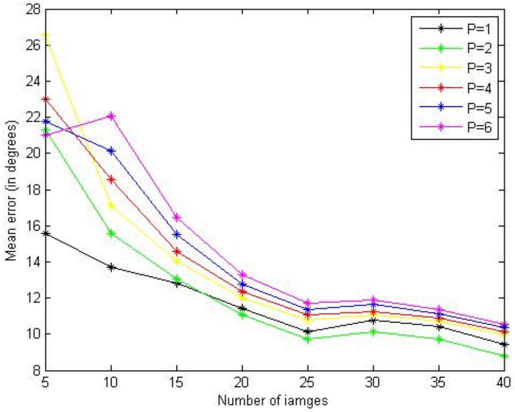
- Mean error of recovery of surface normal with varying number of images for each algorithm
- Noise is 50 percent



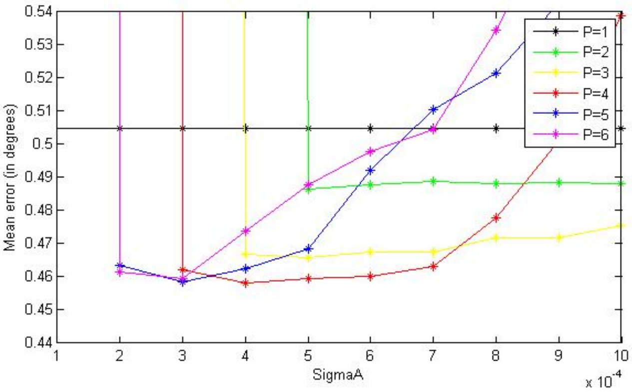
- Mean error of recovery of surface normal with varying amount of additive Gaussian noises for each algorithm
- Number of images is fixed to 40



- Mean error of recovery of surface normal with varying number of images for PL-LS algorithm
- Noise is 10 percent



- Mean error of recovery of surface normal with varying σ_a^2 and p for PL-SBL algorithm
- Number of images is fixed to 40



Overview

Summary

- We explored the basic method of photometric stereo reconstruction
- We improved the performance by introducing sparse regularized algorithms as it could interpret the non-diffuse corruption
- From a different perspective, we implemented a sparse Bayesian learning model to represent the non-diffuse corruption and show state-of-the-art performance
- We extended the sparse Bayesian learning model with piecewise linear model, which could also interpret the non-Lambertian diffuse reflectance and modestly improve the performance

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